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New Sum Rule Determination of the Nucleon Mass

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Abstract

A new QCD calculation of the mass of the nucleon is presented. It makes use of a polynomial kernel in the dispersion integrals tailored to practically eliminate the contribution of the unknown $1/2^+$ and $1/2^-$ continuum. This approach avoids the arbitrariness and other drawbacks attached to the Borel kernel used in previous sum rules calculations. Our method yields stable results for the nucleon mass and coupling for standard values of the condensates. The prediction of the nucleon mass $m_N = (0.945\pm\ .045) GeV$ is in good agreement with experiment.

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1 Introduction

The QCD sum rules method introduced by Shifman et al. [1] has extended the applicability of QCD far beyond simple perturbation theory. The method was adapted to the case of nucleons by Ioffe [2] and independently by Chung, Dosch, Kremer and Schall [3]. These authors showed how to approach one of the fundamental problems of QCD, the calculation of baryon masses from the Lagrangian and the vacuum condensates.

Nucleon mass sum rules start with the correlation function

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0|\eta(x)\eta(0)|0\rangle \tag{1}$$

where η is a nucleon interpolating field constructed from local QCD operators with the quantum numbers of the nucleon. We will choose [2]

$$\eta = e^{abc}(u^a C\gamma^\lambda u^b)\gamma_5\gamma^\lambda d^c \ .$$

which couple maximally to the nucleon. The correlator can be decomposed in terms of invariants,

$$\Pi(q) = q_\mu\gamma^\mu\Pi_1(q^2) + \Pi_2(q^2)$$

with γ_μ standing for the Dirac matrices. $\Pi(t = q^2)$ is an analytic function in the complex t-plane with a pole at $t = m_N^2$ and a cut along the positive real axis starting at $t = (m_N + m_\pi)^2$. The sum rule methods can be traced back to the Cauchy formula

$$\frac{1}{2\pi i}\oint \Pi(t)P(t)\,dt = -\int_0^R \frac{1}{\pi}\mathrm{Im}\Pi(t)P(t)\,dt \tag{2}$$

where the kernel $P(t)$ is an arbitrary analytic function. The integral on the left hand side (l.h.s.) is over a circle of radius R. If R is taken large enough, we can replace $\Pi(t)$ on the l.h.s. by it's QCD and operator product expansion (OPE) counterpart $\Pi_{QCD}(t)$. The right hand side (r.h.s.) involves, apart from the nucleon pole, an integral over the cut, consisting of a background plus a set of nucleonic resonances. Duality means, that the OPE result on the l.h.s. of Eq. (2), is equated to the hadronic contribution on the right hand side. Traditionally the integrand on the r.h.s. is approximated by the "pole plus continuum" model,

$$\frac{1}{\pi}\,\mathrm{Im}\,\Pi(q^2) = \lambda_N^2\delta(q^2 - m_N^2) + \frac{1}{\pi}\theta(q^2 - W^2)\,\mathrm{Im}\,\Pi^{OPE}(q^2) \tag{3}$$

Here m_N is the position of the lowest lying pole with residue λ_N, the coupling of the current to the nucleon state

$$\langle 0|\eta|n\rangle = \lambda\Psi \ ,$$

and an effective continuum threshold W^2 which is determined in the calculation and on which the results depend sensitively.

Most sum rule studies of baryonic currents invoke a Borel transform of the correlator, i.e. they use a kernel

$$P(t) = e^{-t/M^2}$$

which introduces another, more or less arbitrary, parameter providing exponential damping of the continuum(when it is small) and suppressing high dimensional vacuum condensates(when it is large). Stability has to be established under variations of the latter parameters.

The model of Eq.(3) is rather unrealistic in cases with distinct higher resonances. One may consider the resonances explicitly, e.g. for the nucleonic correlator Eq.(1) one can use in the sharp mass approximation

$$\frac{1}{\pi}\,\mathrm{Im}\,\Pi_1(q^2) = \lambda_N^2\ \delta(q^2-m_N^2) + \sum_i \lambda_i^2 \delta(q^2-m_i^2)$$
$$\frac{1}{\pi}\,\mathrm{Im}\,\Pi_2(q^2) = m_N\lambda_N^2\delta(q^2-m_N^2) + \sum m_i\lambda_i^{(+)2}\delta(q^2-m_i^2) - \sum m_i\lambda_i^{(-)2}\delta(q^2-m_i^2)$$

For the nucleonic correlator considered here, there are four radial recurrences $N^+(1440)$, $N^-(1535)$, $N^-(1650)$ and the $N^+(1710)$, where $+$ or$-$ refers to the parity of the state [8] and where the couplings are unknown. The masses and widths of these states are well known experimentally.It is seen from the above that the Borel sum rules involving $\Pi_2(q^2)$ are more reliable than the ones involving $\Pi_1(q^2)$ because of the cancellation between resonances of opposite parities [10]. With so many parameters to vary, the sum rule approach is often viewed more as an art rather than a science.

To overcome these intrinsic ambiguities we have introduced some time ago a sum rule method [7], originally called ACD, which exploits the analyticity properties of the correlator to significantly reduce, in some cases practically eliminate, the contribution of the continuum. The breakthrough in the treatment of the continuum has been the introduction of an integration kernel in the FESR tuned to suppress substantially the resonance energy region above the ground state. This approach, specially adapted to eliminate pronounced resonances, has been recently used to extract very precise values of the light quark masses [11] and condensates [12]. Our approach is based on the fact, that the contribution of the continuum in the integral on the r.h.s. of Eq.(2) arises mostly from the interval

$$2.0GeV^2 \leq t \leq 3.0GeV^2 \tag{4}$$

where the four nucleon resonances lie. This prompts us to choose

$$P(t) = \left(1 - \frac{t}{t_0}\right) \tag{5}$$

where $t_0 = 2.52 GeV^2$ is the location of the common midpoint of the parity even and parity odd resonances. The value of $P(t)$ is small in theregion of the resonances,moreover the contributions of the two parity even resonances $N(1440)$ and $N(1770)$ (or the two parity odd ones $N(1535)$ and $N(1650)$) come with opposite signs, so they tend to cancel in sum rules involving both $\Pi_1(q^2)$and $\Pi_2(q^2)$. A residual model dependency is still unavoidable as inelasticity, non-resonant background and resonance interference are impossible to guess realistically. Also here our approach helps, as a constant background is eliminated by the integration kernel. Having thus minimized the contribution we will neglect it.

The theoretical side of the sum rule, in contrast, is in better shape. The correlator (1) is known including radiative corrections and OPE terms up to dimension d = 9 [2],[4],[5]. There exists the usual uncertainty about the precise values of the condensates and the validity of the factorization assumption used for the higher dimensional condensates. We note that the kernel Eq.(5) will introduce only low dimension condensates into the calculation which are known (or at least estimated).

Apart from the references cited above there are a few more attempts to evaluate the sum rule (2) involving a high sophistication on the theoretical side which is not always commensurate with the primitive model ansatz on the phenomenological side. We therefore think it necessary to present for once an (almost) model independent investigation of the nucleonic sum rule.

2 The calculation

The invariant amplitudes have poles at $t = m_N^2$

$$\Pi_1(t) = \frac{-\lambda_N^2}{(t - m_N^2)} + ...$$
$$\Pi_2(t) = \frac{-m_N.\lambda_N^2}{(t - m_N^2)} + ...$$

Having essentially eliminated the contribution of the continuum, it follows then from Cauchy's theorem that

$$|\lambda|^2 P(m_n^2) = \frac{1}{2\pi i} \oint_{|t|=R} \Pi_1^{QCD} P(t)$$

$$m_N |\lambda|^2 P(m_n^2) = \frac{1}{2\pi i} \oint_{|s|=R} \Pi_2^{QCD} P(t)$$

with

$$(2\pi)^4 \Pi_1^{QCD}(t) = A_0 t^2 \ln\frac{-t}{\mu^2} + A_{01} t^2 \left(\ln\frac{-t}{\mu^2}\right)^2 + A_4 \ln\frac{-t}{\mu^2}$$
$$+ A_6 \frac{1}{t} + +A_{61} \frac{1}{t} \ln\frac{-t}{\mu^2} + A_8 \frac{1}{t^2} + ... \quad (6)$$

and

$$(2\pi)^4 \Pi_2^{QCD}(t) = B_3 t \ln\frac{-t}{\mu^2} + B_7 \frac{1}{t} + B_9 \frac{1}{t^2} + ... \quad (7)$$

The coefficients A_i and B_i are defined as in [5] but for a factor $(2\pi)^4$and powers of t taken explicitly

$$A_0 = -\frac{1}{4}(1 + \frac{71}{12} a), A_{01} = \frac{a}{8}$$
$$A_4 = -\frac{\pi^2}{2} \langle aGG \rangle$$
$$A_6 = -\frac{2}{3}(2\pi)^4 \langle \bar{q}q\bar{q}q \rangle \left(1 - \frac{5}{6} a - \frac{1}{3} a \ln\frac{-t}{\mu^2}\right), A_{61} = \frac{2}{9}(2\pi)^4 \langle qqqq \rangle a$$
$$A_8 = \frac{-1}{6}(2\pi)^4 \mu_0^2 \langle \bar{q}q\bar{q}q \rangle$$
$$B_3 = 4\pi^2 \langle \bar{q}q \rangle \left(1 + \frac{3}{2} a\right)$$
$$B_7 = -\frac{4\pi^4}{3} \langle \bar{q}q \rangle \langle aGG \rangle$$
$$B_9 = (2\pi)^6 \frac{136}{81} a \left\langle (\bar{q}q)^3 \right\rangle$$

where $a = \frac{\alpha_s(\mu^2)}{\pi}$. The terms B_7 and B_9 are given in the factorization approximation. In A_8 we have taken

$$\langle 0 | \bar{q}q\bar{q} a G^c_{\mu\nu} \frac{\lambda_c}{2} \sigma^{\mu\nu} q | 0 \rangle = \mu_0^2 \langle \bar{q}q\bar{q}q \rangle$$

with the parameter $\mu_0^2 = 0.8 GeV^2$ as advocated in [13]. We use the vacuum dominance approximation, also for the four-quark condensate, $\langle \bar{q}q\bar{q}q \rangle \approx \langle \bar{q}q \rangle^2$.

The question at which scale this relation holds is resolved by allowing for generous errors. To avoid the double counting, we keep the logarithmic $\ln(-t/\mu^2)$ contribution in the polarization operator but neglect its anomalous dimension. In any case anamalous dimesion effects are very small [6].

For the finite energy sum rule Eq.(2), we need the known integrals of the form

$$I_{ik} = \frac{1}{2\pi i}\oint dt t^i \left(\ln(-t)\right)^k$$

These are given in convenient form in [14].

With our choice of P(t), we then get

$$(2\pi)^4|\lambda|^2 P(m_N^2) = -A_0 R^3 \left(\frac{1}{3} - \frac{R}{4t_0}\right) - A_{01}R^3(-\frac{2}{9} + \frac{2}{3}\ln(\frac{R}{\mu^2}) + \frac{1}{t_0}(\frac{R}{8}. - \frac{R}{2}\ln(\frac{R}{\mu^2}))$$
$$- A_4 R(1 - \frac{R}{2t_0}) - A_6 - A_{61}(\ln\frac{R}{\mu^2} - \frac{R}{t_0}) + \frac{A_8}{t_0} \qquad (8)$$

$$(2\pi)^4|\lambda|^2 m_n P(m_N^2) = -B_3 R^2 \left(\frac{1}{2} - \frac{R}{3t_0}\right) - B_7 + \frac{B_9}{t_0} \qquad (9)$$

In order to allow the cancellations introduced by $P(t)$ to take place, we choose $R = 2.92\ GeV^2$, i.e. right after the fourth resonance. The nucleon mass is obtained by dividing the two sum rules.

3 Results and concusions

With standard values of the condensates and using the factorization approximation as discussed above, we get a value for the nucleon mass close to the experimental value. For a more precise statement, we need do discuss possible errors in the sum rule. It is important to distinguish two kinds of errors, theoretical and experimental. A theoretical error of $\pm.03GeV$ arises from the uncertainty in the quark condensate. We used $\langle qq\rangle = -(1.90\pm0.14)\times10^{-2}GeV^3$ at $\mu = 2GeV$ corresponding to the limits set by $m_u + m_d = (8.2\pm0.6)MeV$ (at scale $\mu = 2GeV$) [11] in the GMOR relation. Varying the scale parameter μ^2 between $4GeV^2$ and $2GeV^2$ introduces an additional error of $\pm0.03GeV$. Furthermore there is an error due to the strong coupling constant. We use the latest comprehensive update analysis at the τ-scale [15] which gives $\alpha_s(m_\tau^2) = 0.342 \pm 0.01$ and leads to an error of $\pm0.6MeV$ in the nucleon mass. On other theory errors, like the one due to the factorization assumption, we have no quantitative handle, but under reasonable assumptions they turn out to be less important. We estimate the experimental error, by varying t_0, the zero of the polynomial $P(t)$, between $2.36GeV^2$ and $2.72GeV^2$. The resulting error in the nucleon mass is

$$m_N = (945 \pm 4.5)MeV$$

In conclusion, we have presented a simple calculation of the nucleon mass using a kernel in the dispersion integral tailored to minimize the contribution

of the unknown continuum without involving the higher order unknown condensates. Using standard values of the known condensates, we obtain for m_N a value which agrees quite well with the experimental one.

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